FEATURES OF THE FILM FLOW OF LIQUIDS OVER SURFACES WITH REGULAR ROUGHNESS

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The results are presented from an analytical and experimental investigation of the flow of liquid films along vertical surfaces having regular roughness elements.

The widespread use of heat and mass transfer equipment having packed beds in the form of regularly ordered structures is explained by the optimum nature of the conditions in them for carrying out transfer processes and by their low energy consumption. The packed beds of such equipment are formed by grouping into packets vertical sheets with regular surface roughness elements.

The study of the hydrodynamic features of the flow of liquid films over the surfaces of sheet packings is preferably carried out on single-channel models in view of the significant nonuniformities in the distributions of the material streams which unavoidably occur in multichannel structures. The authors have carried out a theoretical and experimental investigation of film flows along vertical surfaces with regular surface roughness elements in the absence of a gas stream, since in the zone of small gas loadings no significant hydrodynamic interaction of the phases is observed.

Let us assume that a thin layer of a viscous liquid is moving along a vertical surface with regular roughness elements, the equation of which is given by the periodic function $x_1 = f_1(z_1)$ (where the z_1 axis is oriented in the direction of $\bar{g}$). It will be assumed that the ratio of $\max f_1(z_1)$ to the mean liquid film thickness is a small quantity ε. The system of equations consisting of the Navier–Stokes equations and the continuity equation which describes the developing film flow over a surface with small heights of the roughness protrusions assumes the following form in terms of dimensionless variable [1]:

$$\alpha^2 Lu = -\frac{\partial P}{\partial x} + \frac{\alpha}{\mathrm{Re}}\,\Delta u, \tag{1}$$

$$Lv = \varepsilon f\,\frac{\partial P}{\partial x} - \frac{\partial P}{\partial z} + \frac{1}{\mathrm{Fr}} + \frac{1}{\alpha\,\mathrm{Re}}\,\Delta v, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} - \varepsilon f'\,\frac{\partial v}{\partial x} = 0. \tag{3}$$

The boundary conditions on the solid substrate and at the free surface of the liquid h(z) are given in the form

$$u = v = 0, \quad x = 0; \quad u = v\,\frac{\partial h}{\partial z}, \quad x = \delta(z). \tag{4}$$

The conditions for the absence of shear stresses and the continuity of the normal stresses at the free surface are represented in the form

$$-4\alpha^2 h'\,\frac{\partial v}{\partial z} + \frac{\partial v}{\partial x} + \alpha^2\,\frac{\partial u}{\partial z} - \varepsilon\alpha^2 f'\,\frac{\partial u}{\partial x} = 0, \quad x = \delta(z), \tag{5}$$

$$-P - \frac{2\alpha}{\mathrm{Re}}\,\frac{1-\alpha^2(h')^2}{1+\alpha^2(h')^2} - \frac{2\alpha}{\mathrm{Re}}\,\frac{h'}{1+\alpha^2(h')^2}\left(\frac{\partial v}{\partial x} + \alpha^2\,\frac{\partial u}{\partial z} - \right. \tag{6}$$

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$$-\varepsilon\alpha^2 f' \frac{\partial u}{\partial x}\right) = -P_r + \frac{Nh''}{[1+\alpha^2 (h')^2]^{3/2}}, \quad x = \delta(z). \tag{6}$$

The following notation is used above for the dimensionless quantities:

$$x = \frac{x_1}{\delta_N} - \varepsilon f(z), \quad z = \frac{z_1}{p}, \quad \alpha = \frac{\delta_N}{p}, \quad \varepsilon = \frac{\max f_1(z_1)}{\delta_N},$$

$$\delta_N = \left(\frac{3\mathrm{Re}\nu^2}{g}\right)^{1/3}, \quad \varepsilon f(z) = \frac{f_1(z_1)}{\delta_N}, \quad u = \frac{u_1 p}{\delta_N v_0}, \quad v = \frac{v_1}{v_0},$$

$$P = \frac{P_1}{\rho v_0^2}, \quad h = \frac{h_1}{\delta_N}, \quad \delta = \frac{\delta_1}{\delta_N} = \frac{h_1 - f_1}{\delta_N}, \quad \mathrm{Re} = \frac{\delta_N v_0}{\nu},$$

$$\mathrm{Fr} = \frac{v_0^2}{gp}, \quad N = \frac{\sigma\delta_N}{\rho p^2 v_0^2}$$

and use is also made of the following operators:

$$L = u\frac{\partial}{\partial x} + v\left(\frac{\partial}{\partial z} - \varepsilon f' \frac{\partial}{\partial x}\right), \quad \Delta = \frac{\partial^2}{\partial x^2} + \alpha^2 \frac{\partial^2}{\partial z^2} - \varepsilon\alpha^2\left(2f'\frac{\partial^2}{\partial x \partial z} + f''\frac{\partial}{\partial x}\right) + \varepsilon^2\alpha^2 (f')^2 \frac{\partial^2}{\partial x^2}.$$

Theoretical analyses of the flow of films over regular substrates have been made by a number of authors. In [1], it is assumed that $\alpha \ll 1$, $\varepsilon \sim 1$, $\mathrm{Re} \sim 1$; in [2], that $\alpha \ll 1$, $\varepsilon \ll 1$, $\mathrm{Re} \sim \varepsilon$; and in [3] that $\alpha \ll 1$, $\varepsilon \sim 1$, $\mathrm{Re} \sim 1/\alpha$.

In our case there are the following ranges of variation of the initial parameters, for example, from the physical conditions for the processes occurring in film-type equipment for heat and mass transfer during evaporative cooling: Re = 500-2000, δ_N = 0.6-1.0 mm, p = 2.5-45 mm, ε = 0.6-0.8, α = 0.02-0.4, i.e., $\alpha^2 \ll 1$, $\varepsilon < 1$, $\alpha\mathrm{Re} \sim 1/\alpha^2$.

In this case, the initial boundary-value problem (1)-(6) without taking into account terms of the second order of smallness with respect to α^2 leads to equations of the boundary-layer type with the known boundary conditions

$$u\frac{\partial v}{\partial x} + v\left(\frac{\partial v}{\partial z} - \varepsilon f' \frac{\partial v}{\partial x}\right) = -\frac{\partial P}{\partial z} + \frac{1}{\mathrm{Fr}} + \frac{1}{\alpha \mathrm{Re}}\frac{\partial^2 v}{\partial x^2},$$

$$\frac{\partial P}{\partial x} = 0, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} - \varepsilon f' \frac{\partial v}{\partial x} = 0, \quad u = v = 0, \quad x = 0, \quad u = v\frac{\partial h}{\partial z},$$

$$x = \delta(z), \quad \frac{\partial v}{\partial x} = 0, \quad x = \delta(z), \quad P = P_g - Nh'', \quad x = \delta(x),$$

so that with respect to the unknown functions v, h, it is found that

$$-\frac{\partial v}{\partial x}\int_0^x \frac{\partial v}{\partial z}\,dx + v\frac{\partial v}{\partial z} = \frac{1}{\mathrm{Fr}} + Nh''' + \frac{1}{\alpha\mathrm{Re}}\frac{\partial^2 v}{\partial x^2}, \tag{7}$$

$$\frac{\partial}{\partial z}\int_0^\delta v\,dx = 0. \tag{8}$$

The function v must satisfy the conditions

$$v = 0, \quad x = 0; \quad \frac{\partial v}{\partial x} = 0, \quad x = \delta; \quad \int_0^\delta v\,dx = 1. \tag{9}$$

It is assumed [4] that the velocity profile v = v(x, z) differs only insignificantly from a parabolic profile, so that

$$v = -\frac{3}{2\delta^3}(x^2 - 2\delta x). \tag{10}$$

Then by carrying out the usual procedure [4] of averaging the first equation for the layer thickness δ for the free surface h(z), the following ordinary differential equation is obtained:

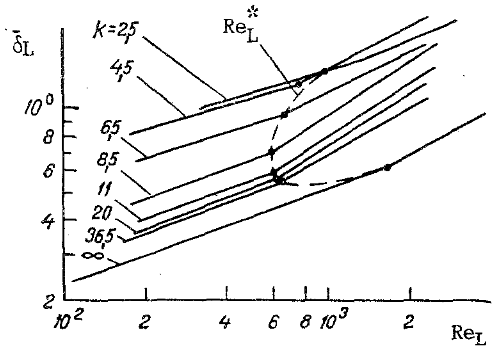

Fig. 1. Effect of the roughness parameter k and the Reynolds number Re on the mean liquid film thickness; Re* is the critical value of the Reynolds number for the liquid film. $\bar{\delta}_L$ is given in mm.

$$(h-\varepsilon f)^3 h''' + a(h'-\varepsilon f') + b(h-\varepsilon f)^3 - b = 0, \tag{11}$$

where a = 6/5 N, b = 3/α NRe.

A periodic solution of Eq. (11) will be sought in the form of the power series

$$h = 1 + \sum_{l=1}^{\infty} h_l \varepsilon^l, \quad \varepsilon < 1.$$

Substitution of the series into the equation leads to an infinite system of linear homogeneous differential equations with constant coefficients:

$$l = 1, \quad h_1''' + ah_1' + 3bh_1 = af' + 3bf, \quad l = m \geqslant 2, \; h_m''' + ah_m' + 3bh_m = \\ = -\sum_{n=1}^{m-1} c_n h_{m-n}''' - \frac{b}{m} \sum_{n=1}^{m-1} (4n-m)\, a_n c_{m-n}, \tag{12}$$

where

$$a_1 = h_1 - f, \quad c_0 = 1, \quad a_m = h_m\,(m \geqslant 2), \quad c_m = \frac{1}{m} \sum_{k=1}^{m} (4k-m)\, a_k c_{m-k}.$$

The right-hand side of each equation of the system (12) can be represented in the form of a Fourier series:

$$\frac{a_{m0}}{2} + \sum_{n=1}^{\infty} (a_{mn} \cos n\omega z + b_{mn} \sin n\omega z), \quad \omega = \frac{2\pi}{p};$$

$$a_{mj} = -\frac{\omega}{\pi} \int_0^p \left[\sum_{n=1}^{m-1} c_n h_{m-n}''' + \frac{b}{m} \sum_{n=1}^{m-1} (4n-m)\, a_n c_{m-n} \right] \cos j\omega z dz,$$

$$j = 0,\ 1,\ 2,\ \ldots;$$

$$b_{mj} = -\frac{\omega}{\pi} \int_0^p \left[\sum_{n=1}^{m-1} c_n h_{m-n}''' + \frac{b}{m} \sum_{n=1}^{m-1} (4n-m)\, a_n c_{m-n} \right] \sin j\omega z dz,$$

$$j = 1,\ 2,\ \ldots,$$

and the solution will also be sought in the form of a Fourier series:

$$h_m = \frac{A_{m0}}{2} + \sum_{n=1}^{\infty} (A_{mn} \cos n\omega z + B_{mn} \sin n\omega z).$$

The coefficients of the series h_m are found from the formula

$$A_{m0} = \frac{a_{m0}}{3b}; \quad A_{mn} = \frac{3ba_{mn} - n\omega(a - n^2\omega^2)\, b_{mn}}{(3b)^2 + n^2\omega^2(a - n^2\omega^2)^2};$$

$$B_{mn} = \frac{3bb_{mn} + n\omega(a - n^2\omega^2)\, a_{mn}}{(3b)^2 + n^2\omega^2(a - n^2\omega^2)^2}.$$

This scheme for solving Eq. (11) has been set up in the form of a program in FORTRAN-IV.

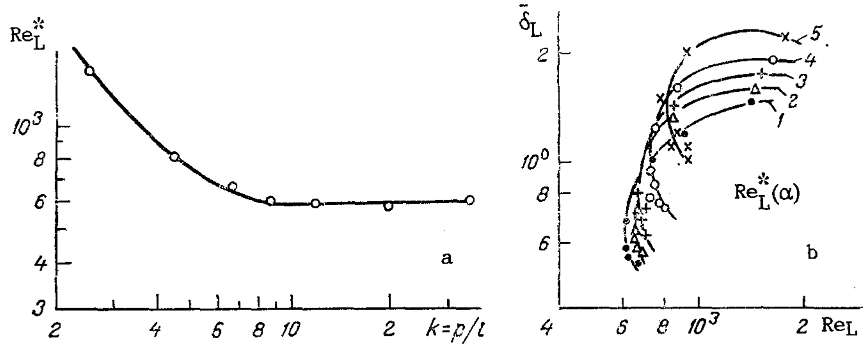

Fig. 2. Relationship for Re* for vertical sheets (a), curve according to Eq. (13), and for inclined sheets (b): 1) slope = 15°; 2) slope = 30°; 3) slope = 45°; 4) slope = 75°.

The hydrodynamics of film flow were studied experimentally on flat sheets with roughness of two types: in the form of horizontal protrusions uniformly distributed over the surface of the sheet characterized by roughness parameters k = p/e (p is the pitch of the roughness, equal to 2.5-45 mm, and e is the height of the protrusions, equal to 1.0 mm), and in the form of fine transverse sinusoidal grooving of height e. In the present investigations the term "smooth" refers to the flat sheets without roughness elements which were used as a reference case for comparison. On the basis of existing recommendations [6, 7] the heights of the protrusions e was taken to be ≈1.0 mm. The problem of the shape of the rib profile of the roughness elements was not considered specially, since according to the data of [8] differences in the shapes of the protrusions (trapezoids, rectangles, triangles) did not have a noticeable effect on the hydrodynamics of the flow, which was determined primarily by the value of k. The effects of the flow regimes were studied, together with the critical value of Re* characterizing the transition phenomenon, the mean liquid film thickness $\bar{\delta}$, and also the presence in the inlet zone of the development of wavy flow and spray formation.

The experiments were carried out by the method of electrical conductivity, which was described earlier in [9] and improved to suit the present investigation. The method makes it possible to carry out measurements in channels of complex shapes. The packing-element sheets were made from plexiglass by means of milling. The sheet dimensions were 250 × 500 mm; the distance between three vertical electrodes of nickel wire was 100 mm; the electrodes were fitted flush-mounted into the surfaces of the sheets. In order to ensure uniform distribution of the liquid film and damping of pulsations, a strip of fine capillary-porous material (Flizelin) was fitted at the upper end of the sheet.

The main results are shown in Fig. 1. A weakly marked wavy flow is observed on the smooth sheets. Beginning with a value of k ≃ 20 (as k is decreased), a monotonic and periodic wavy phenomenon occurs, and the size of the entry zone decreases as the value of k decreases. Simultaneously, a thickening of the film is observed in the range k = 40-4, followed by a thinning for k < 4.

The form of the relationship Re*(k) is interesting. The value of Re* for the smooth plate was ≈1650 (since the flow in the immediate neighborhood of the wall retains its laminar character, the value of Re* is provisional to some extent, and is characterized by a certain range of values). The minimum value of Re* ≃ 600 corresponds to k = 8-12. A further decrease of k leads to an increase of Re*, so that the relationship Re*(k) in the coordinates (Re, $\bar{\delta}$) is described by a complex curvilinear relationship. It may be mentioned that we have obtained a series of analogous curves for inclined sheets for the range of values of the angle of inclination to the vertical of 15-75°. In terms of the coordinates noted above, these curves shift to the right and upward (Fig. 2b). The form of the relationship Re*(k) is explained by the fact that at sufficiently small values of k the roughness ribs are practically joined together, and the conditions of flow of the liquid film approximate to the nature of the flow on a smooth plate without roughness elements. Japanese investigators have also pointed to this [6, 7], noting that the effectiveness of the evaporative cooling of a water film flowing over sheets with regular roughness elements with very small values of k is even lower than in the case of flow over a smooth sheet.

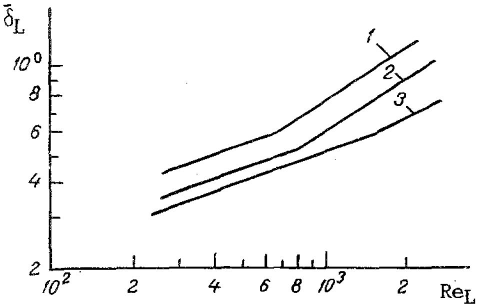

Fig. 3. Effect of the type of roughness on the mean liquid film thickness: 1) protrusions; 2) sinusoids; 3) flat sheet without roughness elements.

The experimental material can be represented in the form of relationships as follows [9]:

$$\bar{\delta} = x_1 \sqrt[3]{\frac{3}{4}\frac{\nu^2}{g}}\,\mathrm{Re}^{x_2} k^{x_3} \exp(x_4 k).$$

The parameters x_1, ..., x_4 are determined by minimization of the object function

$$\varphi = \sum_{i=1}^{n} (\delta^i_{\mathrm{exp}} - \delta^i_{\mathrm{calc}})^2$$

where n is the number of experimental points) by the method of searching with respect to a deformable polyhedron (Nelder–Mid method). The expression

$$\bar{\delta} = 1.375\cdot 10^5 \sqrt[3]{\frac{3}{4}\frac{\nu^2}{g}}\,\mathrm{Re}^{0,165} k^{-0,591} \exp(1{,}44\cdot 10^{-2} k)$$

is obtained for the turbulent flow regime (the mean error in the ranges of values Re = 720-2100 and k = 2.25-33.25 is 6.7%). The boundary between the flow regimes is given by the expression

$$\bar{\delta} = 1.028\cdot 10^4 \sqrt[3]{\frac{3}{4}\frac{\nu^2}{g}}\,\mathrm{Re}^{0,514} k^{-0,327} \exp(3.3\cdot 10^{-3} k)$$

$$\mathrm{Re}^* = -3.36\cdot 10^2 k^{0,661} \exp(-0.039k) + 1650, \tag{13}$$

which describes the experimental data for the vertical sheets (Fig. 2a) with a mean error of 12.1%.

Three characteristic zones of the flow of the liquid films can be distinguished corresponding to the following ranges of the values of the roughness parameter k = p/e:

1) k = 8-12: a stable, regular wavy regime of flow is observed (standing wave model); there is practically no entry zone for the formation of the waves; spray formation is minimal;

2) k < 8: laminar flow occurs; the values of Re^*, $\bar{\delta}$ increase; at sufficiently small values of k the situation practically reverts to flow along a smooth sheet;

3) k > 12: disruptions in the regular wavy flow regime begin; spray formation increases; the entry zone increases; at sufficiently large values of k the situation practically reverts to flow along a smooth sheet.

Thus, the range of values of the roughness parameter of k = 8-12 is of particular interest, where there are optimum conditions for the manifestation of the regular roughness and at the same time the maximum intensity of occurrence of the transfer processes in the liquid film is ensured. It should be stressed that this range of k corresponds in general to its optimum value under the conditions of countercurrent flow of the liquid film and a gas stream.

Figure 3 gives the results obtained on a smooth sheet (curve 3), a sheet with regular roughness elements of the "protrusion" type (curve 1), which was considered above, and a sheet with continuous fine horizontally grooved sinusoidal profiles (curve 2). The values of p and e were approximately the same for the sheets being compared. Strictly speaking, the flow of the liquid film over the sheet with the fine transverse grooving can be regarded as a flow over a continuously curved surface, so that the nature of the flow is little different to that on a smooth sheet. Similar results were obtained in a study of Japanese authors [6], where a wire-wound ribbing with an additional gauze covering was used as the roughness elements.

The form of the relationship $\bar{\delta}(Re)$ in Fig. 3 makes it possible to give preference to roughness elements of the "periodic protrusion" type as the form ensuring the most favorable conditions for intensifying the processes in the film.

The results of calculations carried out on an ES-1035 computer for sheets with grooved sinusoidal profiles were compared with the experimental values, from which it was found that: at Re = 1000, $\bar{\delta}_{calc} = 0.43$, $\bar{\delta}_{exp} = 0.50$; at Re = 2000, $\bar{\delta}_{calc} = 0.57$, $\bar{\delta}_{exp} = 0.64$.

Thus, good agreement is found between the calculated and experimental values of the mean thicknesses of the liquid film.

NOTATION

z_1, x_1, longitudinal and transverse coordinates; v_1, u_1, projections of the velocity on the Z_1, X_1 axes; $x_1 = f_1(z_1)$, the equation of the support surface; $x_1 = h_1(z_1)$, the equation of the free surface of the liquid; g, acceleration of free fall; ν, coefficient of kinematic viscosity; ρ, liquid density; δ_N, thickness of the liquid layer during flow on smooth vertical surface; $\bar{\delta}$, mean thickness of liquid film; v_0, mean velocity of liquid, calculated from flow rate; P_g, pressure on gas side; P, pressure in liquid; σ, surface tension coefficient; p, pitch of roughness elements.

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